INCIDENCE GEOMETRY

Incidence Axiom 1:
For every pair of distinct points P and Q there exists exactly one line l st. both P and Q lie on l.

Incidence Axiom 2:
For every line l there exists at least two distict points P and Q st both P and Q lie on l.

Incidence Axiom 3:
There exists three points that does not lie on any one line

Thrm 2.6.3: If l is any line, then there exsists at least one point P st P does not lie on l.

Thrm 2.6.4: If P is any point, then there are at least two distigt lines l and m st. P lies on both m and l.

Thrm 2.6.5: If l is any line then there exists lines m and n st. l, m, n are distingt lines and both m and n intersects l.

Thrm 2.6.6: If P is any point, then there exists at least one line l st P does not lie on l.

Thrm 2.6.7: There exists three distict lines st no point lie on all three lines.

Thrm 2.6.8: If P is any point, then there exists points Q and R st. P, R and Q are noncollinear

Thrm 2.6.9: If P and Q are two points st $P \neq Q$, then there exists a point R s.t P, Q and R are collinear.

DISTANCE AND RULER POSTULATE

Thrm 3.2.7: If P and Q are any two points, then
1) $PQ = QP$
2) $PQ \geq 0$
3) $PQ = 0$ if, and only if $P = Q$.

Thrm 3.2.16 (The Ruler Replacement Post): For every pair of distinct points P and Q, there is a coordinate function $f: \overleftrightarrow{PQ} \to \mathbb{R}$ st $f(P) = 0$ and $f(Q) > 0$

Thrm 3.2.17 (Betweeness for Points): Let ℓ be a line; let A, B, C be three distinct points that all lie on ℓ; and let $f: \ell \to \mathbb{R}$ be a coordinate function for ℓ. The point C is between A and B if and only if either $f(A) < f(C) < f(B)$ or $f(A) > f(C) > f(B)$

Thrm. 3.2.22 (Existance and Uniqueness of Midpoints): If A and B are distinct points, then there exists a unique point M st M is the midpoint of $\overline{AB}$

Thrm 3.2.23 (Point Construction Post): If A and B are distict points and d is any non-negative real number, then there exists a unique point C st C lies on $\overrightarrow{AB}$ and $AC = d$.

PLANE SEPERATION

Thrm 3.3.4: Let ℓ be a line and let A and B be points that do not lie on ℓ. The points A and B are on the same side of ℓ if and only if $\overline{AB} \cap \ell = \emptyset$. The points A and B are on opposite sides of ℓ if and only if $\overline{AB} \cap \ell = \emptyset$.

Thrm 3.3.9 (The Ray Thrm): Let ℓ be a line, A a point on ℓ and B an external point for ℓ. If C is a point on $\overrightarrow{AB}$ and $C \neq A$, Then B and C are on the same side of ℓ.

Thrm. 3.3.10: Let A, B and C be three non collinear points and let D be a point on the line $\overleftrightarrow{BC}$. The point D is between points B and C if and only if the ray $\overrightarrow{AD}$ is between rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$

Thrm 3.3.12 (Pasch's Axiom) Let $\triangle ABC$ be a triangle and let ℓ be a line st none of A, B and C lies on ℓ. If ℓ intersects $\overline{AB}$, then ℓ also intersects $\overline{AC}$ or $\overline{BC}$

ANGLE MEASURE AND PROTACTOR POST

Lemma 3.4.4: If A, B, C and D are four distict points st C and D are on the same side of $\overleftrightarrow{AB}$ and D is not on $\overleftrightarrow{AC}$, then either C is in the interior of $\measuredangle BAD$ or D is in the interior of $\measuredangle BAC$.

Thrm 3.4.5 (Betweennes Thrm for Rays): Let A, B, C and D be for distict points st. C and D lie on the same side of $\overleftrightarrow{AB}$. Then $\mu(\measuredangle BAD) < \mu(\measuredangle BAC)$ if and only if $\overrightarrow{AD}$ is between rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$

Thrm 3.4.7 (Existance and Uniqueness of Angle Bisectors): If A, B and C are three non collinear points, then there exists a unique angle bisector for $\measuredangle BAC$.

CROSSBAR THRM AND LINEAR PAIR THRM

Thrm 3.5.1 (The Z-thrm): Let ℓ be a line and let A and D be distinct points on ℓ. If B and E are points on opposite sides of ℓ, then $\overrightarrow{AB} \cap \overrightarrow{DE} = \emptyset$.

Thrm 3.5.2 (The Crossbar Thrm): If $\triangle ABC$ is a triangle and D is a point in the interior of $\measuredangle BAC$, then there is a point G s.t. G lies on both $\overrightarrow{AD}$ and $\overrightarrow{BC}$.

Thrm 3.5.3: A point D is in the interior of the angle $\measuredangle BAC$ if and only if the ray $\overrightarrow{AD}$ intersects the interior of the segment $\overline{BC}$

Thrm 3.5.5 (Linear Pair Thrm): If angles $\measuredangle BAD$ and $\measuredangle DAC$ form a linear pair then $\mu(\measuredangle BAD) + \mu(\measuredangle DAC) = 180°$.

Lemma 3.5.7: If $C * A * B$ and D is in the interior of $\measuredangle BAE$, then E is in the interior of $\measuredangle DAC$.

Thrm 3.5.9: If ℓ is a line and P is a point on ℓ, then there exists exactly one line m st P lies on m and $m \perp \ell$.

Thrm 3.5.11 (Existense and Uniqueness of Perpendicular Bisectors) If D and E are two distict points, then there exists a unique perpendicular bisector for $\overline{DE}$

Thrm 3.5.13 (Veritical Angles Thrm): Verticle angles are congurent.

Lemma 3.5.14: Let $[a,b]$ and $[c,d]$ be closed intervals of real numbers and let $f: [a,b] \to [c,d]$ be a function. If f is strictly increasing and onto, then f is continous.

Thrm 3.5.15 (The Continuity Axiom): The function f is described as $f: [0,d] \to [0, \mu(\measuredangle CAB)]$ by $f(x) = \mu(\measuredangle CAD_x)$

SIDE-ANGLE-SIDE POSTULATE

Thrm 3.6.5 (Isosceles Triangle Thrm): The base angles of an isosceles triangle are congruent.

→ If $\triangle ABC$ is a triangle and $\overline{AB} \cong \overline{AC}$, then $\measuredangle ABC \cong \measuredangle ACB$